Ex 3.4 11-13

11. $P(x) = x^3 - 5x^2 + 7x + k$

a $(x-a)^2(x-b) = P(x)$

$(x^2 - 2ax + a^2)(x-b) = P(x)$

$(x^3 - 2ax^2 - bx^2 + 2abx + a^2x - a^2b) = P(x)$

$x^3 = x^3$

$-5x^2 = -2ax^2 - bx^2$

$-5 = -2a - b$ $\qquad$ $b = -2a + 5$

$7x = 2abx + a^2x$

$7 = 2ab + a^2$

$k = -a^2b$

$7 = 2a(-2a+5) + a^2$

$7 = -4a^2 + 10a + a^2$

$3a^2 + 10a + 7 = 0$ $\qquad$ $3 \times 7 = 21$ $\quad$ 3, 7

$3a^2 - 3a - 7a + 7 = 0$

$3a(a-1) + 7(a-1) = 0$

$(3a-7)(a-1) = 0$

$\underline{a = 1}$ or $a = \frac{7}{3}$

as a is an integer

$b = -2(+1) + 5$

$= -2 + 5 =$

$k = -(-1)^2 \times 3$

$= -3$

12. $x(t) = \frac{1}{5}(t^3 - 16t^2 + 55t)$ Grant

$x(t) = \frac{1}{20}(4t^3 - 64t^2 + 207t)$ Lisa

a Grant $x(2) = \frac{1}{5}(8 - 64 + 110)$

$= \frac{54}{5} = 10.8\,m$

Lisa $x(2) = \frac{1}{20}(32 - 256 + 414)$

$= \frac{190}{20} = \frac{19}{2} = 9.5\,m$

b. Grant $x(t) = \frac{t}{5}(t^2 - 16t + 55)$

$= \frac{t}{5}(t-5)(t-11)$

Grant at shore $x(t) = 0$

$0 = \frac{t}{5}(t-5)(t-11)$

$\therefore$ at shore at $t = 0$, $t = 5\,s$ & $t = 11\,s$

first time back to shore

c. Lisa $x(t) = \frac{t}{20}(4t^2 - 64t + 207)$

$= \frac{t}{20}(2t-23)(2t-9)$

Lisa at shore $x(t) = 0$ $\therefore t = 0,\ t = \frac{23}{2},\ t = \frac{9}{2}$

first time

d. Grant completes in 11 s

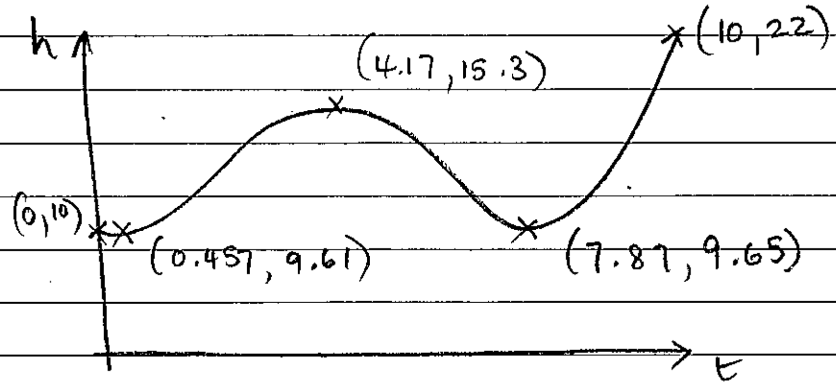
e. Lisa completes in 11.5 s, but runs for 4.5 s

$\therefore$ swims for 7 s

Ex 3.4 Q13

13. $h(t) = 0.03t^4 - 0.5t^3 + 2.3t^2 - 1.8t + 10$

a $0 \leq t \leq 10$



b. $h(t) = 14$ m $\quad t = 2.87302 = 2$ min 52 s

$t = 5.46799 = 5$ min 28 s

$t = 9.24382 = 9$ min 15 s.

c. $h(t) = 10$ m $\quad t = 0,\ t = 0.978553,\ t = 7.38123,\ t = 8.30689$

Solve$(h(t)=10, t)$

first after 5 mins

$t = 7$ mins 23 secs

Ex 3.5 Q 11, 12.

11. $x+3$ is a factor of $P(x) = x^3 + 2x^2 + kx - 6$

a. find k. as $x+3$ is a factor $P(-3) = 0$

$\therefore$ Solve $(P(-3)=0, k)$ or substitute & solve

$$0 = (-3)^3 + 2(-3)^2 + k(-3) - 6$$
$$0 = -27 + 18 - 3k - 6$$
$$0 = -15 - 3k$$
$$3k = -15$$
$$k = -5$$

b.

	1	2	-5	-6
$x+3$	0	3	-3	-6
	1	-1	-2	0

$P(x) = (x+3)(x^2 - x - 2)$

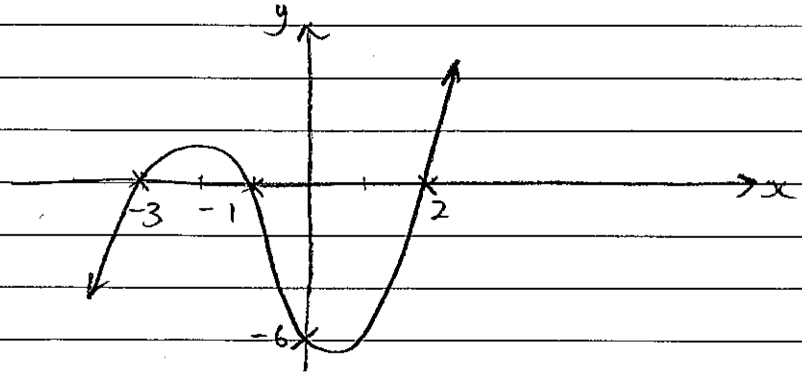
$= (x+3)(x-2)(x+1)$

c. axial intercepts $P(x) = 0$ $x = -3, 2, -1$

$x = 0$ $P(x) = -6$

$(-3,0)(-1,0)(2,0)(0,-6)$

d.



e $P(x) > 0$ "find x values where $y > 0$"

$x \in (-3, -1) \cup (2, \infty)$

$-3 < x < -1 \cup x > 2$

12. $y = a(x-b)^2(x-c)$

tp at $x = 1$ at grd
$\therefore b = 1$

a.

tunnel exits 7m to right of where it commenced
$\therefore c = 7$

so $y = a(x-1)^2(x-7)$

b. at $x = 0$ $y = -1$

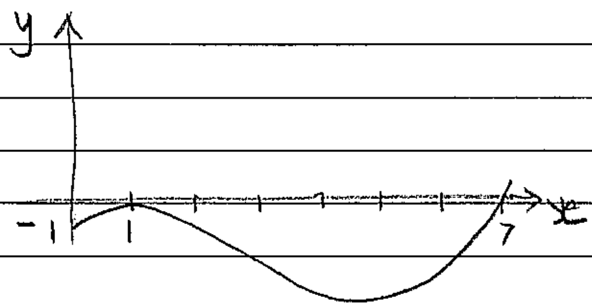
$-1 = a(-1)^2(-7)$

$-1 = -7a$

$a = \frac{1}{7}$

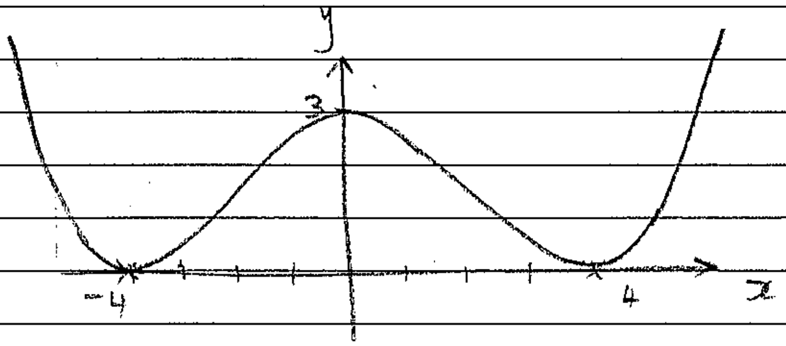
$y = \frac{1}{7}(x-1)^2(x-7)$

d.



c $x = 5$ $y = -4.57$

13a



b. $y = a(x+4)^2(x-4)^2$ sub in $(0,3)$ to find a

$3 = a(4)^2(4)^2$

$3 = 256a$

$a = 3/256$

$y = \frac{3}{256}(x+4)^2(x-4)^2$

c. ramp is 15m wide & symmetrical
$\therefore$ dom $-7.5 \leq x \leq 7.5$

d height at $x = 7.5$ $= 18.99$ m

e. This is very tall/steep. Could be v. dangerous

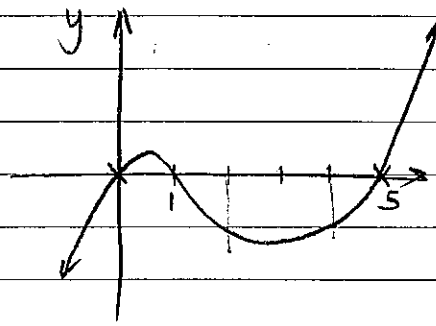
Ex 3.7 Q6, 7

6a. $y = x^3 - 6x^2 + 5x$ $y = x(x^2 - 6x + 5)$

$y = x(x-5)(x-1)$

x int $y = 0$ $x = 0$ $x = 5$ $x = 1$

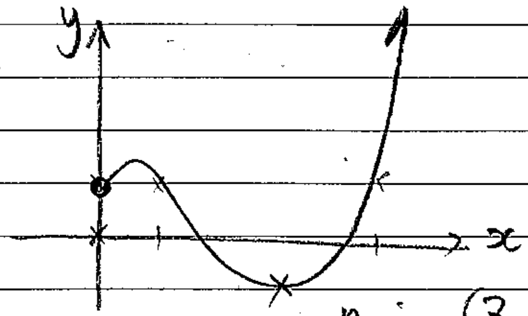
y int $x = 0$ $y = 0$



dom $\in \mathbb{R}$ b.

ran $\in \mathbb{R}$

b.
c.



min $(3.53, -12.1)$

dom $\in R^+$

ran $\in [-12.1, \infty)$

d. $y = x^3 - 6x^2 + 5x$ dom $\in [2, 4)$

$x = 2$ $y = 8 - 24 + 10$
$= -6$

$x = 4$ $y = 64 - 96 + 20$
$= -12$

ran $\in [-13.1, -6)$

7. $P(x) = -0.25x^3 + ax^2 + 21.25x + b$

a. $P(1) = 42$ Solve $(P(1) = 42$ and $P(5) = 132, a, b)$

$P(5) = 132$

$P(1) = 42 = -0.25 + a + 21.25 + b$

$42 = 21 + a + b \quad 21 = a + b \quad b = 21 - a$

$P(5) = 132 = -0.25 \times 125 + 25a + 21.25 \times 5 + b$

$132 = -31.25 + 25a + 106.25 + b$

$57 = 25a + b$

sub $57 = 25a + 21 - a$

$36 = 24a$

$a = 3/2 \quad b = 19\frac{1}{2}$

b. $P(8) = -0.25 \times 8^3 + 1.5 \times 8^2 + 21.25 \times 8 + 19.5$

$= -128 + 96 + 170 + 19.5 = 157.5$

$= 157500$ visitors

c. y dom $\in [0, 12]$ rane $(19.5, 158]$

d. December

e $P(x) < 100$ Solve $(P(x) < 100, x)$